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Thermal Property of a Two-Dimensional Partially Conducting Thin Screen

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Introduction

THE study of the thermal resistance caused by a layered substance imbedded in a medium of different property is important in the design of composite elements. The solution is one dimensional if the layer is uniform and continuous.[1] However, the problem is two dimensional and much more difficult if the layer is composed of discontinuous thin slats arranged in the same plane, such as a two-dimensional screen. This situation occurs in composites that have a layer of evenly spaced thin fiber reinforcements (Fig. 1a).

Since the temperature field is potential, the results of this Note may also be applied to many other analogous problems that are governed by Laplace's equation: electrostatics, magnetostatics, mass diffusion, inviscid fluid flow, and flow in porous media.

There are numerous ways to solve the potential equation, including complex transforms, separation of variables, boundary integrals, summation of singularities, and direct numerical integration. However, as we shall see later, the geometry and the boundary conditions make most methods impractical. We shall use an eigenfunction expansion and collocation method that is well suited for this problem.

Formulation

Let the thermal conductivities of the screen material and the imbedding medium be κ_1 and κ_2, respectively. Let the screen be of thickness t, composed of slats spaced $2L$ apart, and with gap widths of $2bL$. The equivalent conductivity of the screen is determined through its effect on the mean temperature drop under an applied constant flux Q at infinity. The problem is much simplified if we assume the screen is thin, i.e., $t << bL < L$. The modeling is as follows.

Let $T'(x', y')$ be the temperature distribution in which (x', y') are Cartesian coordinates with origin at the midpoint of a gap, where we set T' to zero. Since the screen is thin, we expect the flux to be mainly directly across the slats. Thus, the temperature within the slats is locally linear (except in a small region of order t near the tips). The local heat flux q (for $bL \le y \le L$) is

$$q(y') = \kappa_1[T'[(t/2), y'] - T'[-(t/2), y']]/t \qquad (1)$$

This flux is equated with that from the medium

$$q(y') = \kappa_2 \frac{\partial T'}{\partial x'}\left(\frac{t}{2}, y'\right) = \kappa_2 \frac{\partial T'}{\partial x'}\left(-\frac{t}{2}, y'\right) \qquad (2)$$

Because of symmetry we find

$$T'[(t/2), y'] = -T'[-(t/2), y'] \qquad (3)$$

We normalize all lengths by L, the temperature by QL/κ_2, and drop primes. Equations (1–3) yield

$$\frac{\partial T}{\partial x}\left(\frac{t}{2L}, y\right) = \left(\frac{2L\kappa_1}{t\kappa_2}\right) T\left(\frac{t}{2L}, y\right) \qquad (4)$$

Since $t << L$, Eq. (4) is approximated by

$$\frac{\partial T}{\partial x}(0, y) = \delta T(0, y), \qquad |b| \le |y| \le 1 \qquad (5)$$

where $\delta \equiv (2L\kappa_1)/(t\kappa_2)$ is a nondimensional parameter representing the relative conductivities. We can now solve the decoupled problem for the medium only. The governing equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad (6)$$

with the boundary conditions, Eq. (5). Because of symmetry we have

$$T(0, y) = 0 \qquad |y| < |b| \qquad (7)$$

The condition that the flux at infinity is Q gives

$$\frac{\partial T}{\partial x} = 1 \quad \text{as} \quad x \to \infty \qquad (8)$$

Because of periodicity we can consider only the domain $x \ge 0$, $|y| \le 1$ (Fig. 1b), with the additional condition

$$\frac{\partial T}{\partial y} = 0 \quad \text{on} \quad y = \pm 1 \qquad (9)$$

Nonconducting Case

In this special case, $\delta = 0$, and the problem is analogous to the potential flow over a screen. A Schwarz–Christoffel transform[2] yields

$$T = \frac{2}{\pi} Re\left(\cosh^{-1}\left\{\frac{-i\sinh[(\pi/2)z]}{\sin[(\pi/2)b]}\right\}\right) \qquad (10)$$

where $z = x + iy$. The temperature along the x axis is

$$T(x, 0) = \frac{2}{\pi} Re\left[\ell n\left(\frac{-i\sinh[(\pi/2)x]}{\sin[(\pi/2)b]} + \sqrt{-\left\{\frac{\sinh[(\pi/2)x]}{\sin[(\pi/2)b]}\right\}^2 - 1}\right)\right] \sim x + \frac{2}{\pi}\ell n\left[\frac{1}{\sin(b\pi/2)}\right] + 0(e^{-\pi x}), \qquad x \to \infty \qquad (11)$$

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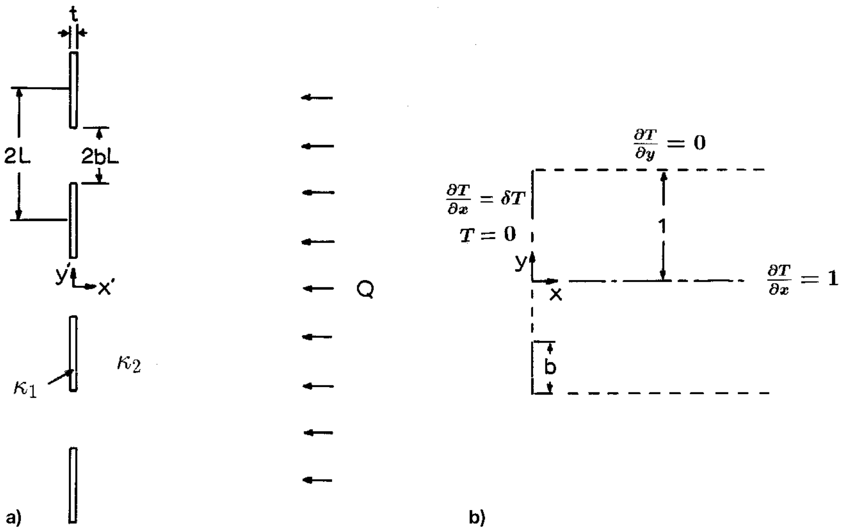

Fig. 1 a) Cross section of the screen and b) solution domain with boundary conditions.

Thus, the screen is equivalent to a solid plate, which causes a normalized temperature drop of

$$D = (4/\pi)\ell n[1/\sin(b\pi/2)] \tag{12}$$

We use D as the measure of equivalent normalized resistance of the screen. The equivalent conductivity is $\kappa_2 t/(LD)$.

Partially Conducting Case

When $\delta \neq 0$, complex transform is no longer useful. This is because the boundary condition, Eq. (5), does not transform into a simple relationship. Since T is even in y, the general solution to Eq. (6) and satisfying Eqs. (8) and (9) is

$$T(x, y) = x + A_0 + \sum_{n=1}^{\infty} A_n \cos(\alpha y)e^{-\alpha x} \tag{13}$$

where $\alpha \equiv n\pi$, and constants A_i are to be determined. The mixed conditions [Eqs. (5) and (7)] at $x = 0$, give

$$1 - \sum_1 \alpha A_n \cos(\alpha y) = \delta \left[A_0 + \sum_1 A_n \cos(\alpha y)\right], \quad b < y \leq 1 \tag{14}$$

$$A_0 + \sum A_n \cos \alpha y = 0, \qquad 0 \leq y < b \tag{15}$$

We choose N evenly spaced points in $0 \leq y \leq 1$:

$$y_j = (j - 1)/N, \qquad j = 1 \quad \text{to} \quad N + 1 \tag{16}$$

and truncate the infinite series to N terms. Then, Eqs. (14) and (15) become

$$\delta A_0 + \sum_{n=1}^{N} (\delta + \alpha)\cos(\alpha y_j)A_n = 1, \quad j = k + 1 \quad \text{to} \quad N + 1 \tag{17}$$

$$A_0 + \sum_{n=1}^{N} \cos(\alpha y_j)A_n = 0, \quad j = 0 \quad \text{to} \quad k \tag{18}$$

where $k = \text{integer}[Nb]$ is the number of points in the gap. Equations (17) and (18) represent $N + 1$ equations and $N + 1$ unknowns A_i. These linear algebraic equations are easily inverted. The accuracy is ascertained by increasing N. In general, the error is less than 2% for $N \approx 80$.

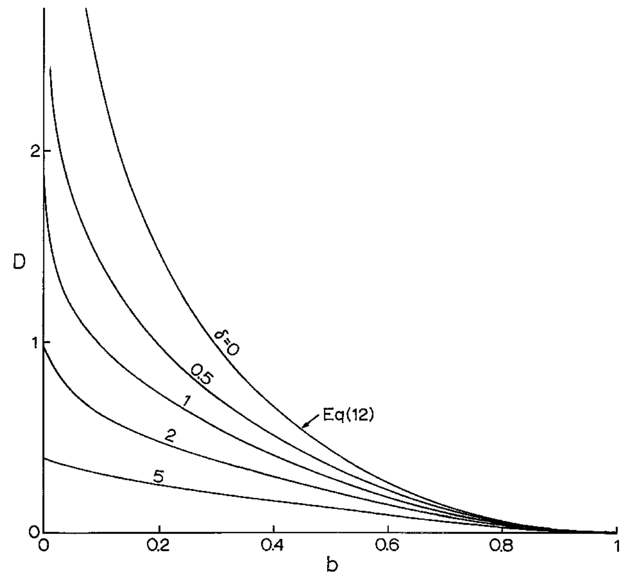

Fig. 2 D for various b and δ. The curve for $\delta = 0$ is from Eq. (12).

Results

The normalized temperature drop caused by the introduction of the screen is

$$D = 2 \lim_{x\to\infty} [T(x, y) - x] = 2A_0 \tag{19}$$

In other words, the screen is equivalent to a solid plate of the same thickness with a conductivity of

$$\bar{\kappa} = (2/\delta D)\kappa_1 \tag{20}$$

In the limit when $\delta = 0$, as in the nonconducting case, $\bar{\kappa} = (t/LD)\kappa_2$. The effect of gap width ratio b is contained in D. Figure 2 shows the normalized temperature drop or equivalent resistance D as a function of b for various constant screen property δ. The value of D is $2/\delta$ at $b = 0$, then drops to zero as b is increased to 1. Note the change in D is much larger for small gap widths.

Discussions

Assuming the screen is thin, we have reduced the effects of the slats into a boundary condition, Eq. (5). The difficult con-